Name ____ Date __________

Algebra 2H Unit 11: Sequences and Series Notes: FILLED IN VERSION

Sequences

In Common Core Algebra I, you studied **sequences**, which are ordered lists of numbers. A **sequence** is formally defined as **a function that has as its domain the set the set of positive integers**, i.e. $\{1, 2, 3, ..., n\}$.

Example 1 A sequence is defined by the equation $a(n) = 2n - 1$.

(a) Find the first three terms of this sequence, denoted by a_1, a_2, and a_3. *HINT: Substitute n in as 1 to get the first term, etc.*

$a_1 = 2(1) – 1 = 2 – 1 = 1$

$a_2 = 2(2) – 1 = 4 – 1 = 3$

$a_3 = 2(3) – 1 = 6 – 1 = 5$

(b) Which term has a value of 53? *HINT: Either continue the pattern or set the equation a(n) equal to 53.*

$a_n = 2(n) – 1 = 53$

$2n – 1 = 53$

$2n = 54$

$n = 27$ So, the 27^{th} term would be 53.

(c) Explain why there will not be a term that has a value of 70.

All of the term values are odd…since 2 times anything would result in an even number, then subtracting one from that would always leave you an odd answer.

Example 2 Recall that sequences can also be described by using **recursive definitions**. When a sequence is defined *recursively*, terms are found by operations on previous terms. Remember, $f(n)$ means the nth term and $f(n-1)$ means the previous term.

A sequence is defined by the recursive formula: $f(n) = f(n-1) + 5$ with $f(1) = -2$.

(a) Generate the first five terms of this sequence. Label each term with proper **function** notation.

$f(1) = -2$ $f(2) = f(1) + 5 = -2 + 5 = 3$

$f(2) = 3$ $f(3) = f(2) + 5 = 3 + 5 = 8$

$f(3) = 8$ $f(4) = f(3) + 5 = 8 + 5 = 13$

$f(4) = 13$ $f(5) = f(4) + 5 = 13 + 5 = 18$

So, the first five terms are: {-2, 3, 8, 13, 18}. Don't forget to include the given first term of -2 in your answer.

(b) Determine the value of $f(20)$. *HINT: Think about how many times you have added 5 to –2.*

So, to get from f (1) to f (20), there have been 19 changes of 5. Mathematically it would look like this:

$f(20) = f(1) + 19(5) = -2 + 95 = 93$ So, $f(20) = 93$.

Example 3 Determine a recursive definition, in terms of $f(n)$, for the sequence shown below. Be sure to include a starting value. *HINT:* $f(n-1)$ *represents the previous term. Find the pattern and apply it to the previous term.*

5, 10, 20, 40, 80, 160, …

$f(n) = 2 * f(n-1)$ Since it seems to be doubling each time.

Example 4 For the recursively defined sequence $t_n = (t_{n-1})^2 + 2$ and $t_1 = 2$, the value of t_4 is

(1) 18
(2) 38
(3) 456
(4) 1446

$t_2 = (t_1)^2 + 2 = (2)^2 + 2 = 6$
$t_3 = (t_2)^2 + 2 = (6)^2 + 2 = 38$
$t_4 = (t_3)^2 + 2 = (38)^2 + 2 = 1446$

Example 5 One of the most well-known sequences is the Fibonacci, which is defined recursively using two previous terms. Its definition is given below.

$f(n) = f(n-1) + f(n-2)$ and $f(1) = 1$ and $f(2) = 1$

Generate values for: $f(3)$, $f(4)$, $f(5)$, and $f(6)$ (in other words, the next four terms of this sequence).

$f(3) = f(1) + f(2) = 1 + 1 = 2$
$f(4) = f(2) + f(3) = 1 + 2 = 3$
$f(5) = f(3) + f(4) = 2 + 3 = 5$
$f(6) = f(4) + f(5) = 3 + 5 = 8$

Example 6 It is often possible to find algebraic formulas for simple sequences, and this skill should be practiced. Find an algebraic formula $a(n)$, similar to that in Example 1, for each of the following sequences. *This formula is referred to as the* ***explicit formula***. Explicit formulas use the index (the term's position). Think: "How can I use the term's position to create the term itself?" Be sure your formula works for all terms. Recall that the domain (n) that you map from will be the set $\{1, 2, 3, ..., n\}$.

(a) 4, 5, 6, 7, ...

$a(n) = a_n + 3$ Since each position # had three added to it.

(b) 2, 4, 8,16, ...

$a(n) = 2^n$ Since each # is a power of 2.

(c) 10, 15, 20, 25, …

$a(n) = 5n + 5$ A bit tougher…but notice the pattern…needed two parts to make it work!

(d) -1, 1, -1, 1, ...

$a(n) = (-1)^n$ Self-explanatory (?)

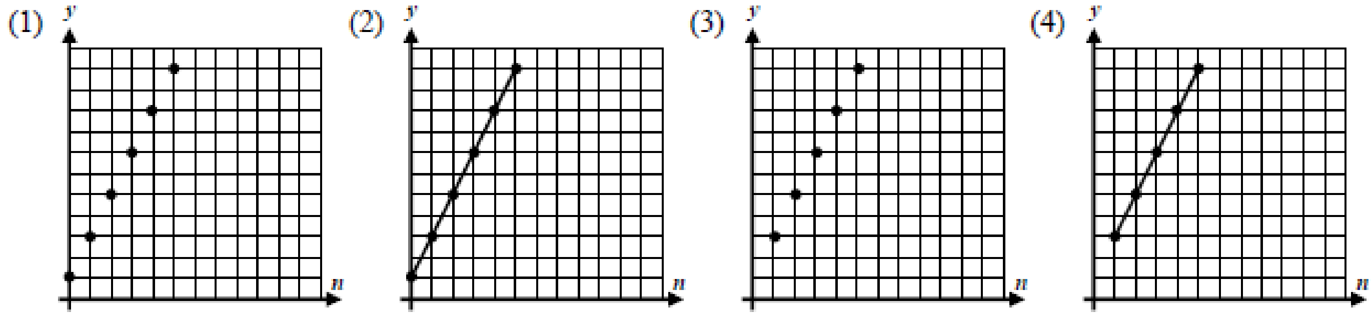
(e) $5, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \ldots$

$a(n) = \frac{a_n}{n}$ Each is a fraction with the term position as the denom.

(f) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

$a(n) = \frac{1}{n^2}$ Notice that the denominator is the square of the position number?

Example 7 Which of the following would represent the graph of the sequence $a_n = 2n+1$? Explain your choice. Remember, the domain is the set of positive integers, i.e. $\{1, 2, 3, ..., n\}$.



Explanation: It would be…choice #3! If we are only using the set of positive integers, then the first range value would be 3, not 1, since 0 isn't in the domain. And although it behaves like a line in y = mx + b form, it's not a line, per se. Plus, choice #4 is a line segment, and the domain is infinite.

Arithmetic and Geometric Sequences

In Common Core Algebra I, you studied two particular sequences known as **arithmetic** (based on constant addition to get the next term) and **geometric** (based on constant multiplying to get the next term). In this lesson, we will review the basics of these two sequences.

Arithmetic Sequence Recursive Definition

Given $f(1)$, $f(n) = f(n-1)+d$ or given a_1 then $a_n = a_{n-1}+d$

where d is called the **common difference** and can be positive or negative.

Remember, $f(n)$ or a_n means the nth term, and $f(n-1)$ or a_{n-1} means the previous term.
d, the common difference, is the difference between any two consecutive terms.
d is the pattern; if there is a subtraction pattern, d is negative.

Example 8 Generate the next three terms of the given arithmetic sequence. Why is this an arithmetic sequence?

$$f(n) = f(n-1) + 6 \text{ with } f(1) = 2$$

It's arithmetic because you're adding the same number, 6, each time.

Next three terms would be: 8, 14, 20

Example 9 For some number t, the first three terms of an arithmetic sequence are $2t$, $5t - 1$, and $6t + 2$. What is the numerical value of the fourth term? *HINT: Since this is an arithmetic sequence, whenever you subtract two consecutive terms, you should always get the same common difference. Set up an equation that will solve for t.*

$(6t + 2) - (5t - 1) = (5t - 1) - 2t$

$t + 3 = 3t - 1$

$2 = 4t$

$2 = t$

Difference between any 2 consec terms must be constant. So, we make an equation using the difference between pairs of terms, and set them equal. Solving shows us that t = 2

So, substituting t = 2, the first term is 4, then 9, then 14…notice the pattern is increasing by 5 each time? So the common difference (d) is 5. Thus, our 4th term would be 19.

*It is important to be able to determine a general term of an arithmetic sequence based on the value of the index variable (the subscript). Remember this formula is referred to as the **explicit formula**. The next exercise walks you through the thinking process involved.*

Example 10 Consider $a_n = a_{n-1} + 3$ with $a_1 = 5$.

(a) Determine the value of $a_2, a_3,$ and a_4.

$a_2 = 8$, $a_3 = 11$, $a_4 = 14$ Added 3 each time.

(b) How many times was 3 added to 5 in order to produce a_4?

We added 3 three times.

(c) Use your result from part (b) to quickly find the value of a_{50}.

If we apply the same idea, we would have to add 3 forty-nine times to get from the first Term (a_1) to the 50th term (a_{50}). Therefore, it would look like this…

$a_1 + 49(3) = 5 + 147 = 152 \leftarrow$ This is the 50th term.

(d) Write a formula for the nth term of an arithmetic sequence, a_n , based on the first term, a_1 , d and n. *This formula is the explicit formula for an arithmetic sequence and can be found on the reference sheet.*

To find any term then, we just need to know how many times we're adding d to the starting #.

$a_n = a_1 + d(n-1)$ You always multiply by 1 less than the term you're looking for.

Example 11 Given that $a_1 = 6$ and $a_4 = 18$ are members of an arithmetic sequence, determine the value of a_{20}. *HINT: First use the explicit formula for* a_4 *and solve for d. Then use it again to find* a_{20}.

Using the formula we just found…

$a_4 = a_1 + d(3)$ Since there were 3 changes to get from the first term to the fourth term.

$18 = 6 + 3d$ Now, solve for d.

$12 = 3d$

$4 = d$

So, now to find term number 20 (a_{20}). We know that there are 19 changes from the first term to the 20^{th} term.

$a_{20} = a_1 + 19(4)$

$a_{20} = 6 + 19(4)$

$a_{20} = 82$

Geometric sequences are defined very similarly to arithmetic, but with a multiplicative constant.

Geometric Sequence Recursive Definition

Given $f(1)$, $f(n) = f(n-1)\cdot r$ or given a_1 then $a_n = a_{n-1}\cdot r$

where r is called the **common ratio,** and can be positive or negative and is often a fraction.

Remember, $f(n)$ or a_n means the *n*th term, and $f(n-1)$ or a_{n-1} means the previous term.
r, the common ratio, is the ratio between any two consecutive terms.
r is the pattern; if there is a division pattern, *r* is a fraction.

Example 12 Generate the next three terms of the geometric sequences given below.

(a) $a_1 = 4$ and $r = 2$

Remember that the r acts as a multiplier…$a_1 = 4$, $a_2 = 8$, $a_3 = 16$, $a_4 = 32$

(b) $f(n) = f(n-1)\cdot\frac{1}{3}$ with $f(1) = 9$

So, $f(1) = 9$, $f(2) = 3$, $f(3) = 1$, $f(4) = 1/3$

Like arithmetic, we also need to be able to determine any given term of a geometric sequence based on the first value, the common ratio, and the index. *Again, this formula is referred to as the* ***explicit formula.***

Example 13 Consider $a_1 = 2$ and $a_n = a_{n-1} \cdot 3$.

(a) Generate the value of a_4.

$a_1 = 2$, $a_2 = 6$, $a_3 = 16$, $a_4 = 54$

(b) How many times did you need to multiply 2 by 3 to find a_4. What is that the same as?

We had to multiply three times to get from the first term to the fourth term. Since this is repeated multiplication, it follows the same patterns as an exponential! (See how we tie in previous units???)

(c) Use your result from part (b) to quickly determine the value of a_{10}.

So, to get from the first term to the tenth term, we'd need to multiply 3 nine times. Using what we know about exponentials and repeated multiplication, we can do it faster than if we just kept hitting "times 3" on the calculator. That would work, but if the terms are very far apart, you could waste valuable time doing that.

$a_{10} = a_1 * 3^9 = 2 * 19683 = 39366$

(d) Write a formula for the *n*th term of a geometric sequence, a_n, based on the first term, a_1, r and n. *This formula is the explicit formula for a geometric sequence and can be found on the reference sheet.*

$a_n = a_1 * r^{n-1}$ As noted above, this formula is given to you on the NYS reference sheet.

Summation Notation

Much of our work in this unit will concern **adding the terms of a sequence**. In order to specify this addition or summarize it, we introduce a new notation, known as **summation or sigma notation** that will represent these sums. This notation will also be used later in the course when we want to write formulas used in statistics.

Summation (Sigma) Notation

$$\sum_{i=a}^{n} f(i) = f(a) + f(a+1) + f(a+2) + ... + f(n)$$

where ***i*** is called the **index variable**, which starts at a value of *a*, ends at a value of *n*, and moves by unit increments (increase by 1 each time). In other words, you are adding all the terms created by substituting in the numbers from $i = a$ to $i = n$ into $f(i)$.

Example 14 Evaluate each of the following sums **without** the use of your calculator.

(a) $\sum_{i=3}^{5} 2i$

2(3) + 2(4) + 2(5) =
6 + 8 + 10 = 24

(b) $\sum_{k=-1}^{3} k^2$

$(-1)^2 + (0)^2 + (1)^2 + (2)^2 + (3)^2 =$
1 + 0 + 1 + 4 + 9 = 15

(c) $\sum_{j=-2}^{0} 2^j$

$(2)^{-2} + (2)^{-1} + (2)^0 =$
¼ + ½ + 1 = 7/4

Note that you are simply substituting numbers into a given formula. You begin with the # on the bottom of the Sigma, and use every integer until you reach the # on top of the Sigma.

Example 15 Evaluate each of the following sums **with** the use of your calculator. (***Templates Menu – be sure the expression is being typed correctly.***) Can be done without calc, as well.

(a) $\sum_{i=1}^{5} (-1)^i$

$(-1)^1 + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 =$

-1 + 1 + -1 + 1 + -1 = -1

(b) $\sum_{k=0}^{2} (2k+1)$

(2(0) +1) + (2(1) + 1) + (2(2) +1) =

(1) + (3) + (5) = 9

(c) $\sum_{i=1}^{3} (i(i+1))$

(1(1 + 1)) + (2(2 +1)) + (3(3+1)) =

(2) + (6) + (12) = 20

Example 16 Which of the following represents the value of $\sum_{i=1}^{4} \frac{1}{i}$?

(1) $\frac{1}{10}$

(2) $\frac{9}{4}$

(3) $\frac{25}{12}$ → $= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{12}{12} + \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{25}{12}$

(4) $\frac{31}{24}$

Example 17 Consider the sequence defined by $a_n = a_{n-1} + 2a_{n-2}$ and $a_1 = 0$ and $a_2 = 1$. Find the value of $\sum_{i=4}^{7} a_i$. *HINT: The summation is telling you to add the 4th through the 7th terms of this sequence together. First find the terms using the given recursive definition.*

Note that this is asking you to use two different ideas. First, use the idea of a recursive sequence, where each term depends on the one before it (or two terms before it in this case), then find their sum. We need to find the 4th through the 7th terms before we can add them up.

$a_3 = a_2 + 2(a_1) = 0 + 1 = 1$

$a_4 = a_3 + 2(a_2) = 1 + 2(1) = 3$

$a_5 = a_4 + 2(a_3) = 3 + 2(1) = 5$

$a_6 = a_5 + 2(a_4) = 5 + 2(3) = 11$

$a_7 = a_6 + 2(a_5) = 11 + 2(5) = 21$

So…add terms 4 through 7 now…3 + 5 + 11 + 21 = 38

Example 18 It is also good to be able to place sums into sigma notation.
Express each sum using sigma notation. Use *i* as your index variable. First, consider any patterns you notice amongst the terms involved in the sum. Then, work to put these patterns into a formula and then a summation.

(a) 9 + 16 + 25 + … + 100

All the numbers seem to be perfect squares…so:

$\sum_{3}^{10}(i^2)$

(b) -6 + -3 + 0 + 3 + … + 15

They seem to be increasing by 3, but we need to find a pattern that will produce all the terms in the series…

$\sum_{-2}^{5}(3i)$

I apologize for how my computer shows the Sigma notation…not much I can do about that. Note that for (a) we go from 3 to 10. You can start at any integer, and go up to any other integer. But you must use every integer from the bottom number up to the top number. For (b), each number was a multiple of 3, so we just needed to figure out where to start.

Example 19 Which of the following represents the sum 3 + 6 + 12 + 24 + 48 ?

(1) $\sum_{i=1}^{5} 3^i$

(2) $\sum_{i=0}^{4} 3(2)^i$

(3) $\sum_{i=0}^{4} 6^{i-1}$

(4) $\sum_{i=3}^{48} i$

Note that 3 of the choices work for the first term, but only #2 works to produce each term in the series.

Series

A **series** is simply the **sum of the terms of a sequence**.

If the set $\{a_1, a_2, a_{3,\ldots}\}$ represent the elements of a sequence then the series, S_n, is defined by:

$$S_n = \sum_{i=1}^{n} a_i$$

Anytime you were evaluating a summation problem, you were evaluating a series.

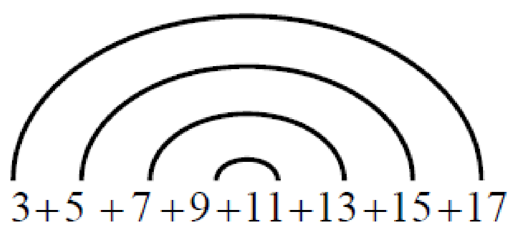
Example 20 Given the arithmetic sequence defined by $a_1 = -2$ and $a_n = a_{n-1} + 5$, then which of the following is the value of $S_5 = \sum_{i=1}^{5} a_i$? *HINT: This is asking you to find the sum of the first 5 terms.*

(1) 32 (3) 25
(2) 40 (4) 27

You can do this without the calc, just find the first 5 terms and then add them up. They are: -2, 3, 8, 13, 18 → 40

The sums associated with arithmetic sequences, known as ***arithmetic series****, have interesting properties, many applications and values that can be predicted with what is commonly known as* ***rainbow addition****.*

Example 21 Consider the arithmetic sequence defined by $a_1 = 3$ and $a_n = a_{n-1} + 2$. The series, based on the first eight terms of this sequence, is shown below. Terms have been paired off as shown.



(a) What does each of the paired off sums equal?
20

(b) Why does it make sense that this sum is constant?
You're increasing/decreasing from each end by a constant value (2)

(c) How many of these pairs are there?
4 pairs

(d) Using your answers to (a) and (c) find the value of the sum.
4 pairs of 20 would = 80

(e) In general, each pair will have the sum of the first and last terms, $a_1 + a_n$. The number of pairs will always be half the number of total elements, or $\frac{n}{2}$. So, does it make sense our sum will always be given by $S_n = \frac{n}{2}(a_1 + a_n)$? Use this formula to verify your answer in part d.

$$S_8 = \frac{8}{2}(3 + 17) = 4(20) = 80$$

This formula is one of the ones **NOT** on the reference sheet…not sure why.

Sum of an Arithmetic Series

Given an arithmetic series with n terms, $\{a_1, a_2, ..., a_n\}$, then its sum is given by: $S_n = \frac{n}{2}(a_1 + a_n)$

Example 22 Which of the following is the sum of the first 100 natural numbers? Show the process that leads to your choice.

(1) 5,000 (3) 10,000 $S_{100} = \frac{100}{2}(1 + 100) = 50(101) = 5050$

(2) 5,100 (4) 5,050

Example 23 Find the sum of each arithmetic series described or shown below using the formula. (Remember the formula on the reference sheet for an arithmetic sequence: $a_n = a_1 + (n-1)d$.)

(a) The sum of the sixteen terms given by: $-10 + -6 + -2 + \ldots + 46 + 50$.

$S_{16} = \frac{16}{2}(-10 + 50) = 8(40) = 320$ We had all the needed information to solve this one!

(b) The first term is -8, the common difference, d, is 6 and there are 20 terms.

To solve this one, we need to first find the value of the 20th term (the last term).

$a_{20} = -8 + (19)(6) = -8 + 114 = 106$ Now, we can use this value for a_{20}

$S_{20} = \frac{20}{2}(-8 + 106) = 10(98) = 980$

(c) The last term is $a_{12} = -29$ and the common difference, d, is -3.

This time, we need to work backwards and find the 1st term (a_1) of the sequence.

$a_{12} = a_1 + (11)(-3) \rightarrow -29 = a_1 + (-33) \rightarrow$ Therefore, $a_1 = 4$

$S_{12} = \frac{12}{2}\left(4 + (-29)\right) = 6(-25) = -150$

(d) The sum $5 + 8 + 11 + \ldots + 77$.

So, we know the first and last terms, and we know that d = 3, but we don't know how many terms there are. So, we'll work backwards and find the number of terms.

$a_n = a_1 + (n-1)d$ This is our general equation.

$77 = 5 + (n-1)3$ Sub in the values we do know, then solve for n.

$77 = 5 + 3n - 3$

$77 = 3n + 2 \rightarrow 75 = 3n \rightarrow 25 = n$ So, there are 25 total terms. Plug into formula

$S_{25} = \frac{25}{2}(5 + 77) = 12.5(82) = 1025$ A little more work, but nothing we can't handle!

Example 24 Kirk has set up a college savings account for his son, Maxwell. If Kirk deposits \$100 per month in an account, increasing the amount he deposits by \$10 per month each month, then how much will be in the account after 10 years? *HINT: 10 years → 120 months*

First, we need to find out how much is deposited in month #120 (the last month), to solve.

$a_{120} = 100 + 119(10) = 100 + 1190 = 1290$

$S_{120} = \frac{120}{2}(100 + 1290) = 60(1390) = 83400$ Good for Maxwell!

Geometric Series

Just as we can sum the terms of an arithmetic sequence to generate an arithmetic series, we can also sum the terms of a geometric sequence to generate a **geometric series**.

Example 25 Given a geometric series defined by the recursive formula $a_1 = 3$ and $a_n = a_{n-1} \cdot 2$, which of the following is the value of $S_5 = \sum_{i=1}^{5} a_i$? *HINT: This is asking you to find the sum of the first 5 terms.*

(1) 106
(2) 75
(3) 93
(4) 35

So, find the next four terms in the sequence

$a_1 = 3$ (given), $a_2 = 6$, $a_3 = 12$, $a_4 = 24$, $a_5 = 48$
So the sum would be: 3+6+12+24+48=93

The sum of a finite number of geometric sequence terms is less obvious than that for an arithmetic series, so it is given to you on the ***reference sheet***.

Sum of a Finite Geometric Series

For a geometric series with *n* terms defined by its first term, a_1, and its common ratio, *r*, the sum is given by:

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

Example 26 Which of the following represents the sum of a geometric series with 8 terms whose first term is 3 and whose common ratio is 4? Show the process that leads to your choice.

(1) 32,756
(2) 28,765
(3) 42,560
(4) 65,535

$$S_8 = \frac{3-3(4^8)}{1-4} = \frac{3-3(65536)}{-3} = \frac{3-196608}{-3} = \frac{-196605}{-3}$$

This one is all about using the given formula. It is on the reference sheet, so no need to memorize.

Example 27 Find the value of the geometric series shown below. Show the calculations that lead to your final answer. *HINT: Find r then use the formula,* $a_n = a_1 \cdot r^{n-1}$*, to find the number of terms, n.*

$6 + 12 + 24 + \ldots + 768$ We must first find how many terms there are. It should be obvious that r = 2.

$768 = 6 * 2^{n-1}$ Now, solve for n. Divide out the 6, then use common bases.

$128 = 2^{n-1}$ 128 is a power of 2.

$2^7 = 2^{n-1}$ Now, set the powers equal, and solve. Thus, n = 8.

So: $S_8 = \frac{6-6(2^8)}{1-2} = \frac{6-6(256)}{-1} = \frac{6-1536}{-1} = \frac{-1530}{-1} = 1530$

Example 28 Electra earns $33,000 in her first year of teaching and earns a 4% increase in each successive year.

(a) Write a geometric series formula, S_n, for Electra's total earnings over n years.

$$S_n = \frac{33000-33000(1.04)^n}{1-1.04}$$

(b) Use this formula to find Electra's total earnings for her first 15 years of teaching, to the nearest cent.

$$S_{15} = \frac{33000-33000(1.04)^{15}}{1-1.04} = \$660{,}778.39$$

Example 29 A person places 1 penny in a piggy bank on the first day of the month, 2 pennies on the second day, 4 pennies on the third, and so on. Will this person be a millionaire at the end of a 31-day month? Show the calculations that lead to your answer.

$$S_{31} = \frac{1-1(2)^{31}}{1-2} = 2{,}147{,}483{,}648\ pennies = \$21{,}147{,}836.48$$

So, yes, definitely a millionaire!!!

Example 30 Devin began running a month ago to get back in shape. The first day he ran 0.5 miles. Each day after that he ran 10% more than the previous day for a total of 30 days. Use the formula for the sum a geometric series to calculate the total distance Devin ran over the 30 days. Round to the nearest thousandth of a mile.

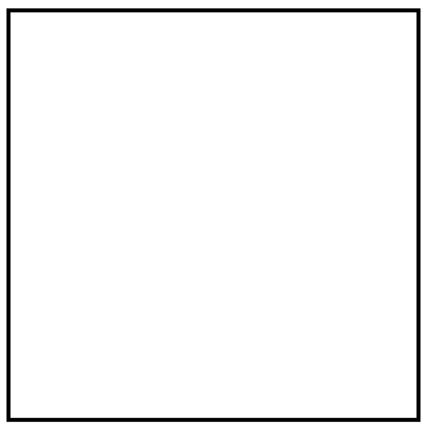
$$S_{30} = \frac{.5-.5(1.1)^{30}}{1-1.1} = 82{,}247\ miles$$

WE ARE GOING TO SKIP THIS PAGE…IT WAS AN "EXTRA" TOPIC FOR US TO DO IF TIME ALLOWED!

Sum of an Infinite Geometric Series

Geometric series in which $|r| < 1$ will have finite answers for their infinite sums. This can be easily demonstrated in a geometric manner for the specific case of: $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = 1$. (Ex. – See Unit square below)

Using the sum of a finite geometric series, $S_n = \frac{a_1(r^n - 1)}{r-1}$ and the fact that as ***n*** approaches infinity, r^n approaches 0, it can be shown that $S_\infty = \frac{a_1}{1-r}$. In the example of the unit square, $a_1 = \frac{1}{2}$ and $r = \frac{1}{2}$, so according to the formula, the sum is ½ divided by ½ or $S_\infty = 1$.



Example 31 What is the sum of the infinite geometric series whose first term is 180 and whose third term is 20?

Example 32 What is the sum of the infinite geometric series whose first term is 120 and whose fourth term is -15?